TN310_PT8_cracked_deflection_3
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IMPLEMENTATION OF DEFLECTION WITH ALLOWANCE FOR FLEXURAL CRACKING IN ADAPT-PT

This Technical Note details the implementation of instantaneous deflection of concrete members with due allowance for flexural cracking in ADAPT-PT software. The work is based on the relationships recommended in ACI-318 [ACI-318, 2008].

1 DEFLECTIONS

Deflections are calculated by ADAPT-PT due to dead loading, post-tensioning, live loading, and combinations of foregoing creep. Both elastic precracking and post-cracking conditions are treated.

1.1 Overview

In many instances, concrete members crack under service load. Cracking reduces the flexural stiffness of a member. As a result, for the same loading, a cracked concrete member deflects more than if its sections were uncracked. A common practice is to determine the loss of stiffness in a member due to cracking and base the deflection calculation on a "reduced moment of inertia, Ie" when the applied moment at a section exceeds the cracking capacity of the same section.

1.2 Equivalent Moment of Inertia

The post-cracking reduced moment of inertia is represented through an Equivalent Moment of Inertia, I_e. The variation of the equivalent moment of inertia for a simply supported beam, in which the applied moment exceeds the cracking moment of the section, is shown in the schematic of **Fig. 1-1**. The equivalent moment of inertia calculated by the program is given by [ACI-318, 2008]:

$$I_e = (M_{cr} / M_a)^3\ I_g + [1-(M_{cr} / M_a)^3]I_{cr} \leq I_g \tag{1.2-1}$$

Where,

Icr	=	Moment of inertia of cracked section;
Ie	=	Effective moment of inertia;
Ma	=	Applied moment at the section where cracking occurs; and,
Mcr	=	Moment that initiates cracking of section.

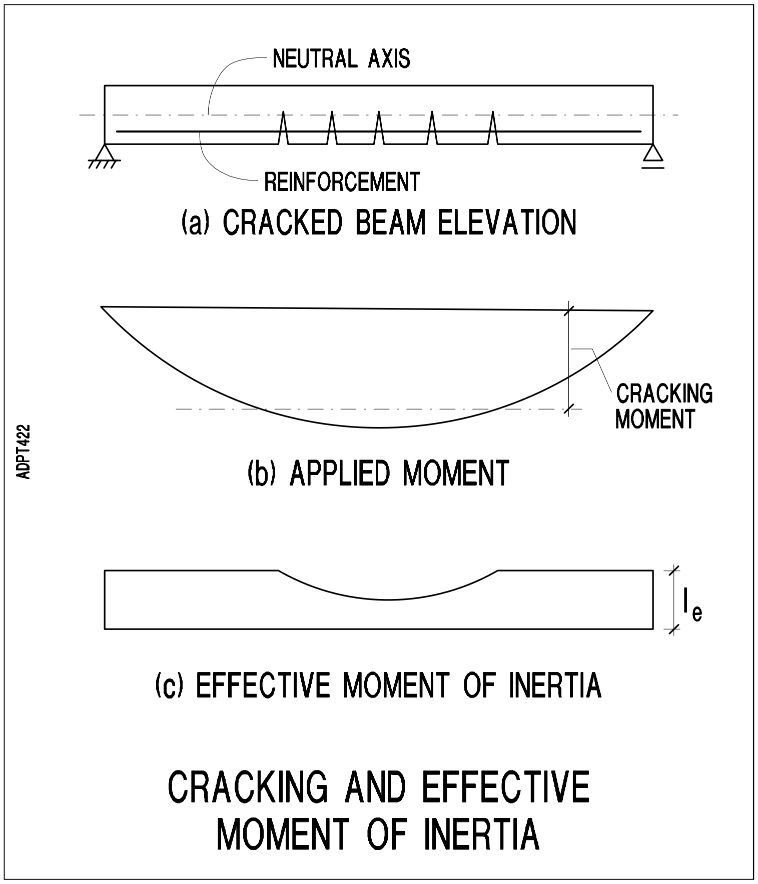
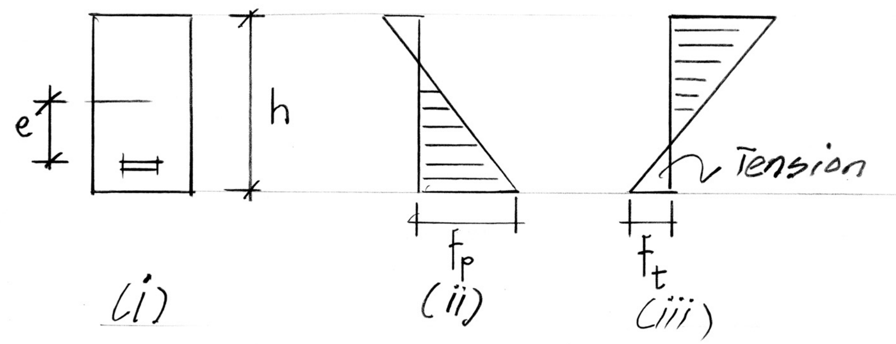


FIGURE 1.2-1

The applied moment, M_a, is calculated using elastic theory and gross moment of inertia for the uncracked section – gross moment of inertia, I_g. The change in distribution of moment in indeterminate structures as a result of cracking in concrete generally leads to a reduction in the initially calculated cracked deflection based on the elastic values of Ma. Conservatively, this reduction in deflection is not accounted for in ADAPT-PT.

Cracking moment, M_{cr} , is calculated using the following relationship(1.2-2), which is a function of section geometry, applied axial force (P) and tensile strength of concrete (f_t). The externally applied moment that would initiate cracking of the section (Mcr) must eliminate the initial compressive stress fp (decompression) and cause a tensile stress ft (cracking stress). **Fig.1.2-2** shows the schematic of a section reinforced with a post-tensioned tendon.



(i) Generic section with post-tensioning tendons at distance "e" from centroid
(ii) Distribution of stress due to post-tensioning only
(iii) Distribution of stress at initiation of crack

POST-TENSIONED SECTION AND DISTRIBUTION OF STRESS

FIGURE 1.2-2

$$Mcr = S^*(fp + ft) \tag{1.2-2}$$

Where, S = section modulus

The stress due to post-tensioning is given by:

$$fp = P/A + Mpt^*c/I$$

Where,

Mpt = moment due to post-tensioning [= P*e + hyperstatic moment]
P = effective force due to post-tensioning;
A = area of section;
c = distance of farthest tension fiber under service load from the centroid; and
I = second moment of area of the section.

ft = tensile strength of the concrete. Program considers this as the allowable stress entered by the user for the final condition.

In its first release of ADAPT-PT, the hyperstatic component of the post-tensioning moment is not included in the reduction of stiffness due to cracking.

Values of I_g are based on the geometry of the concrete cross section, without accounting for the amount and location of reinforcement. For the common case of rectangular and flanged sections (**Fig. 1.2-3**), these values are:

For rectangular section:

$$I_g = bd^3/12 \tag{1.2-3}$$

For flanged section:

$$I_g = h_f^3(b - b_w)/12 + b_w h^3/12 + h_f(b - b_w)(h - (h_f/2) - y_t)^2 + b_w h(y_t - h/2)^2 \quad (1.2\text{-}4)$$

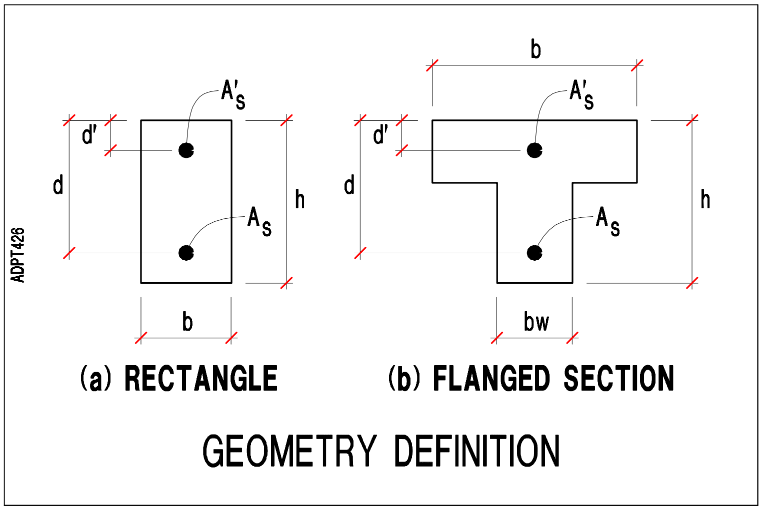
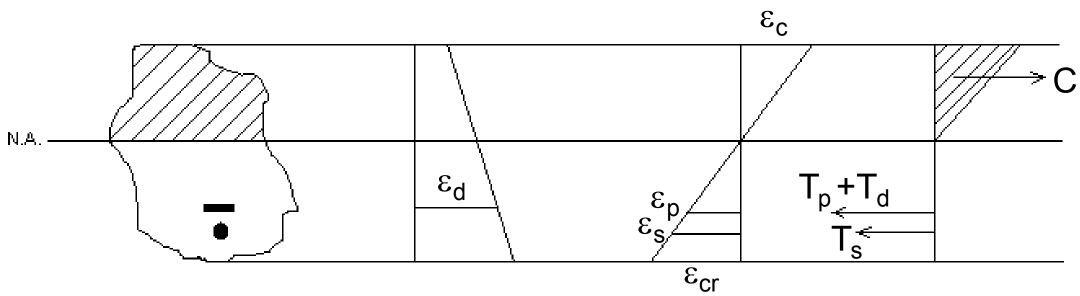


FIGURE 1.2-3

The cracked moment of inertia depends on the strain and force distributions on the cross-section illustrated in **Fig. 1.2-4**, where concrete is assumed to take no tension. Using the assumption of (i) plane sections remain plane, and (ii) the equilibrium consideration of tensile force on section equals compression force, the position of the neutral axis, kd, can be determined. For any section with tensile reinforcement and post-tensioning, the procedure is as follows:

For sections under prestressing forces, under the application of the cracking moment Mcr, the tensile forces on a section consist of the contribution of nonprestressed reinforcement (Ts), and prestressing tendons. However, the tensile force from prestressing tendons can be broken down to the part that reduces the initial compressive stress in concrete at the level of prestressing to zero (decompression Td) and the part (Tp) that brings the stress at the farthest tension fiber to ft

1. Tension force in rebar (T_s)
2. Tension in prestressing reinforcement (T_p)
3. Tension due to decompression (T_d)



FORCE COMPATIBILITY IN A GENERAL PT SECTION

FIGURE 1.2-4

Location of neutral axis is obtained by trial and error to satisfy the force equilibrium shown in **Fig.1.2-4**.

Tension force in rebar (T_s) and tendon (T_p) are calculated based on the strain and modulus of elasticity.

To calculate decompression force (T_d), the strain at concrete at the level of tendon is calculated (ε_d). The decompression force is the force required to make zero strain.

$$\varepsilon_d = \frac{1}{E_c}\left(\frac{P}{A} + \frac{M_{pt} * e}{I_g}\right)$$

In its first release, ADAPT-PT uses the following relationship, where the contribution of the hyperstatic moment in the strain ed is disregarded.

$$\varepsilon_d = \frac{1}{E_c}\left(\frac{P}{A} + \frac{Pe^2}{I_g}\right) \qquad (1.2\text{-}5)$$

where:

P is the force in tendon
A is the area of the section
I_g is the gross moment of inertia
e is the distance from the centroid of the tendon to centroid of concrete section
E_c is the modulus of elasticity of concrete

Then, the force (Td) is simply obtained by:

$$T_d = \varepsilon_d E_s A_p \qquad (1.2\text{-}6)$$

where:

E_s is the modulus of elasticity of tendon
A_p is the area of tendon

Once the position of the neutral axis is obtained, moment of inertia for both RC and PT section are calculated by use of the equivalent concrete section for all mild steels and prestressing steels, the area is replaced by the actual area multiplied by the modulus of elasticity of steel to concrete. Transformed sections after cracking for RC section is given in **Fig.1.2-5**.

The computed moment of inertia for rectangular and T-sections are:

For rectangular section:

i. Without compression rebar:

$$Icr = b(kd)^3/3 + nAs\ (d-kd)^2 + nAps\ (d-kd)^2 \qquad (1.2\text{-}7)$$

Where,

kd is the neutral axis obtained from strain compatibility .

ii. With compression rebar:

$$Icr = b(kd)^3/3 + nAs\ (d-kd)^2 + nAps\ (d-kd)^2 + A_S'(n-1)(kd-d')^2 \qquad (1.2\text{-}8)$$

For flanged section with compression zone exceeding the flange thickness:

i. Without compression rebar:

$$Icr = h_f^3\ (b - b_W)/12 + b_W\ (kd)^3/3 + h_f\ (b - b_W)(kd - h_f/2)^2 + nA_S(d - kd)^2 + nA_{ps}(d - kd)^2 \qquad (1.2\text{-}9)$$

ii. With compression rebar:

$$I_{cr} = h_f^3\ (b - b_W)/12 + (b_W\ (kd)^3)/3 + h_f\ (b - b_W)(kd - h_f/2)^2 + nA_S(d - kd)^2 + A_S'(n - 1)(kd-d')^2 + nA_{ps}(d - kd)^2 \qquad (1.2\text{-}10)$$

For other sections, a similar procedure is used.

In the above calculations, program conservatively considers As as the required rebar from the serviceability and strength design. If the user wants to get the deflection based on the provided rebar, user can enter the provided rebar as the base reinforcement.

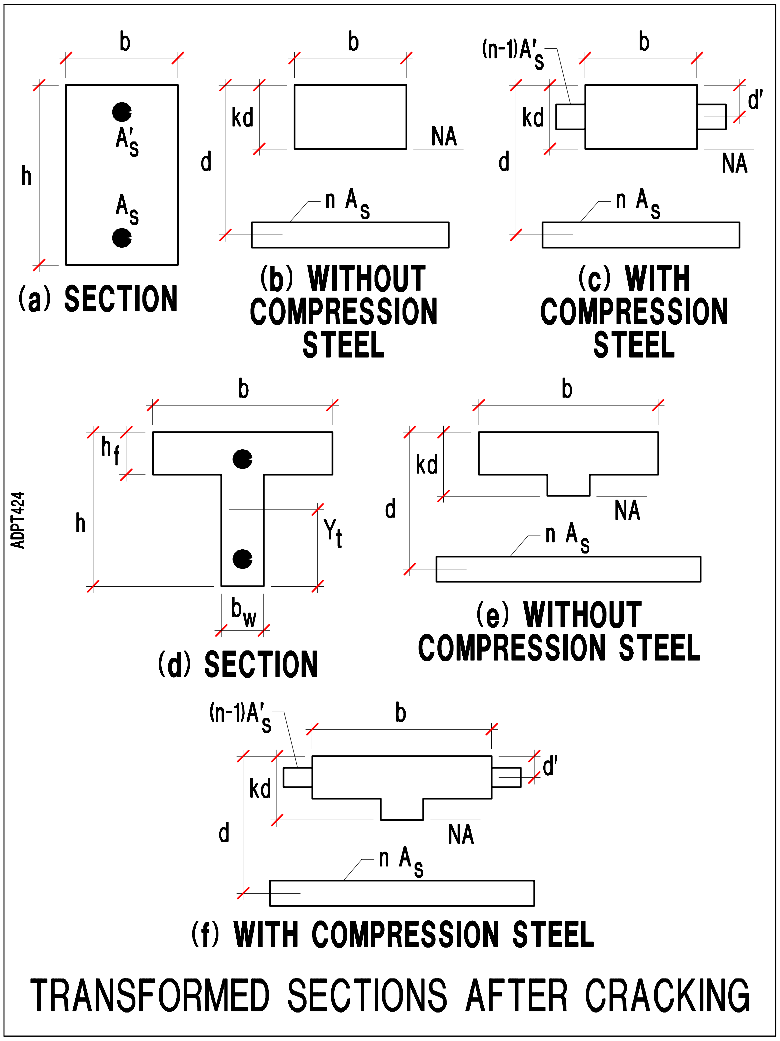


FIGURE 1.2-5

REFERENCES

ACI 318, (2008)